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(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2015

FIRST YEAR

Date : 25/05/2015 **MATHEMATICS FOR ECO (General)**

Time : 11 am – 2 pm **Paper : II** Full Marks : 75

[Use separate Answer Book for each group]

Group – A

Answer ***any five*** questions from the following : (5 × 7)

1. (a) Examine whether $\underset{x\to 0}{Lt}\sin x$ exist. 4

   (b) If $y = x^{2n}$, where n is a positive integer, then prove that $y_n = 2^n \; 1.3.5.\cdots(2n-1) \; .x^n$. 3

2. (a) Give an example of a function $f:\mathbb{R}\to\mathbb{R}$ which is continuous on $\mathbb{R}$ but not differentiable on $\mathbb{R}$. 3

   (b) Evaluate $\lim_{x\to+\infty}\left(1+\frac{e}{x}\right)^{\frac{x}{2}}$. 4

3. (a) Show by $\in-\delta$ definition that the function

   $$f(x) = x^2\sin\frac{1}{x},\; x\neq 0$$
   $$= \quad 0,\; x=0$$

   Is continuous at $x = 0$. 4

   (b) Show that if a function $f:\mathbb{R}\to\mathbb{R}$ is differentiable on $\mathbb{R}$ then it must be continuous on $\mathbb{R}$. 3

4. (a) State Cauchy's Mean Value theorem & interpret the result geometrically. 4

   (b) Verify Rolle's theorem in the function $f(x) = \cos^2 x$ in $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$. 3

5. (a) Can we apply L' Hospital's Rule in case of the limit $\underset{x\to 0}{Lt}\frac{\sin x}{x}$? Give reason. 3

   (b) Using M.V.T. prove that $0 < \frac{1}{x}\log\left(\frac{e^x-1}{x}\right) < 1$. 4

6. (a) Show that : $a^x = 1 + x\log a + \frac{x^2}{2!}\;\log a\;^2 + \cdots\cdots + \frac{x^{n-1}}{(n-1)!}\;\log a\;^{n-1} + \frac{x^n}{n!}a^{\theta n}\;\log a\;^n$. where $a>0 \;\& 0<\theta<1$. 3

   (b) Using Maclaurin's series expansion prove that, $\sin^{-1}x = x + \frac{1}{2}.\frac{x^3}{3} + \frac{1}{2}.\frac{3}{4}.\frac{x^5}{5} + \cdots\infty$ $-1\le x\le 1$. 4

7. (a) Define and give an example of a monotone increasing function. 3

   (b) If $a>b>0$ & $f(\theta) = \frac{(a^2-b^2)\cos\theta}{a-b\sin\theta}$, then find the maximum value of $f(\theta)$. 4

8. (a) Prove that the function defined by: $f(x) = 3|x| + 4|x-1|$, $\forall x\in R$ has a minimum value 3 at $x = 1$. 2

   (b) A window of fixed perimeter (including the base of the arc) is in the form of a rectangle surmounted by a semicircle. The semi-circular portion is fitted with coloured glass while the rectangular portion is fitted with clear glass. The clear glass transmits three times as much light per square meter as the coloured glass does. What is the ratio of the sides of the rectangle so that the window transmits the maximum light? 5

Group – B

Answer ***any four*** questions from the following: (4 × 10)

9. (a) Let s be the set of all 2 x 2 real symmetric matrices. Check whether s is a subspace of the vector space of all 2 x 2 real matrices. 6

(b) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by $S = \{(x,y,z)\in\mathbb{R}^3 : x+2y=z,\ 2x+3z=y\}$ 4

10. (a) Find a basis for the vector space $\mathbb{R}^3$, containing the vectors (1,0,1) and (1,1,1). 5

(b) Prove that the intersection of two subspaces of a vector space V over a field F is a subspace of V. 5

11. (a) Determine the conditions for which the system

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

admits of (i) only one solution,
(ii) no solution
(iii) many solutions. 6

(b) Solve the system of equations: 4

$$x_1+3x_2+x_3=0$$
$$2x_1-x_2+x_3=0$$

12. (a) Define Linear dependence and independence in case of a set of vectors. 2

(b) Let A be a set of vectors which contains the null vector. Check whether the vectors in A is linearly independent. Give reasons. 3

(c) Define a Linear Transformation $T:\vee\to\vee$, where $\vee$ is a finite dimensional vector space. Show that if $T^{-1}:\vee\to\vee$ exist then it is also a linear transformation. 5

13. (a) Prove that the Linear Transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x+y,y+z,z+x),\ (x,y,z)\in\mathbb{R}^3$ is non-singular. Determine T^{-1}. 5

(b) Let V be the vector space of all real polynomials of degree ≤ 3 and $T:\vee\to\vee$ be defined by $T(p(x)) = x\dfrac{d}{dx}(p(x)),\ \ p(x)\in\vee$. Determine the matrix of T relative to the ordered basis $\{1,x,x^2,x^3\}$. 5

14. (a) Determine the Linear Transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ which maps the basis vectors (0,1,1), (1,0,1), (1,1,0) of $\mathbb{R}^3$ to the vectors (2,0,0), (0,2,0), (0,0,2) respectively. Verify that $\dim\ker T+\dim\operatorname{Im}T=3$. 6

(b) Let $M_{2\times2}(\mathbb{R})$ be a vector space of all real 2 x 2 matrices. Consider a Linear Transformation $T:M_{2\times2}(\mathbb{R})\to M_{2\times2}(\mathbb{R})$ defined by $T(A)=A^T$ check whether T is invertible? 4

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